Counting Data
Part 2: Understanding Combinatorics

INFO-1301, Quantitative Reasoning 1

University of Colorado Boulder

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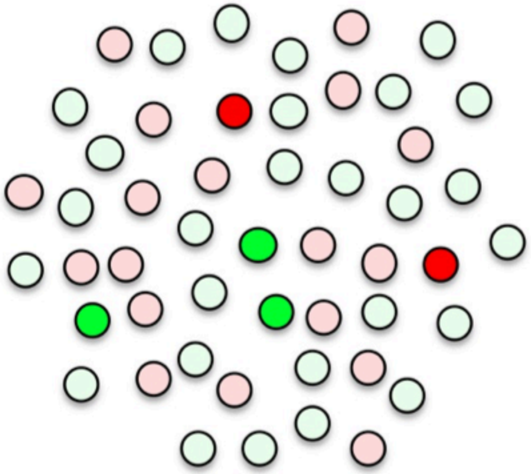
Example 1: Sampling

Suppose you want to know how many people in a group of 50 are unemployed, so you sample 5.

How many combinations of 5 people could you have chosen?

“50 choose 5” = 2,118,760



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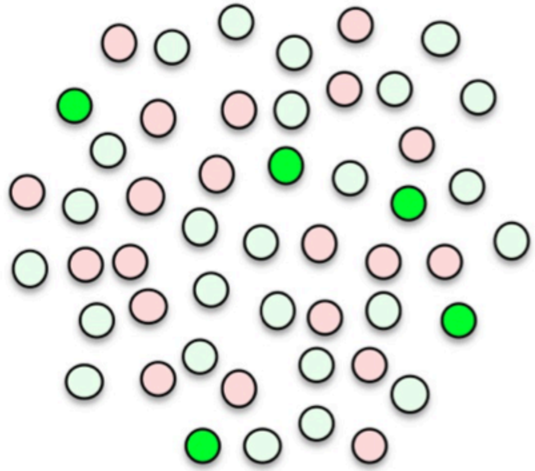

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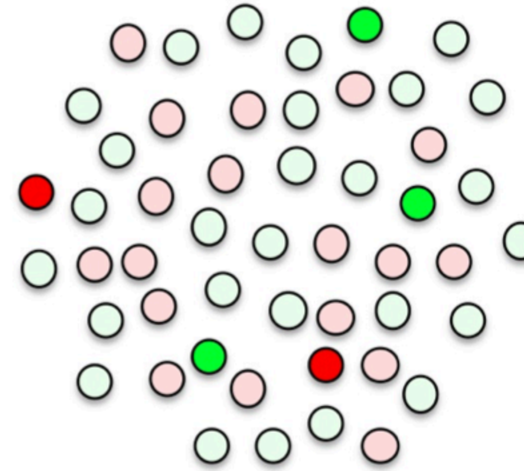

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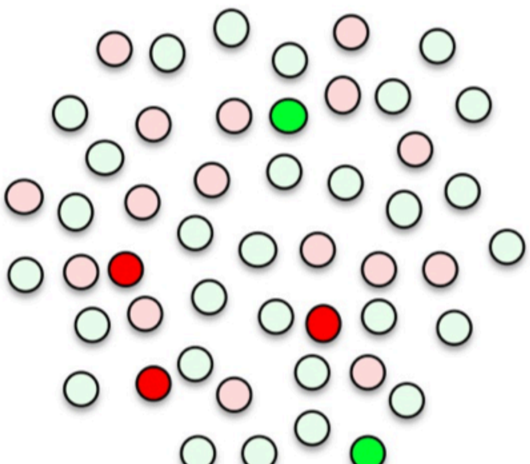

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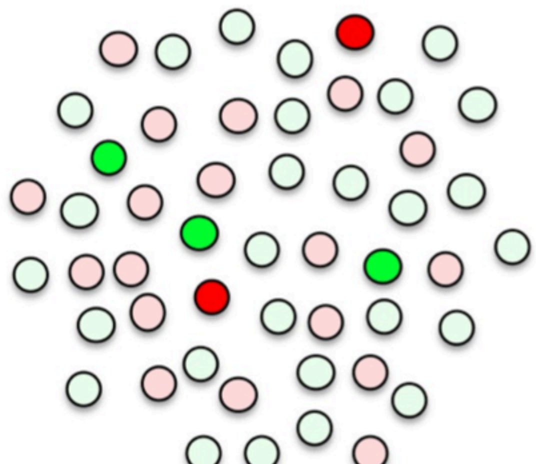

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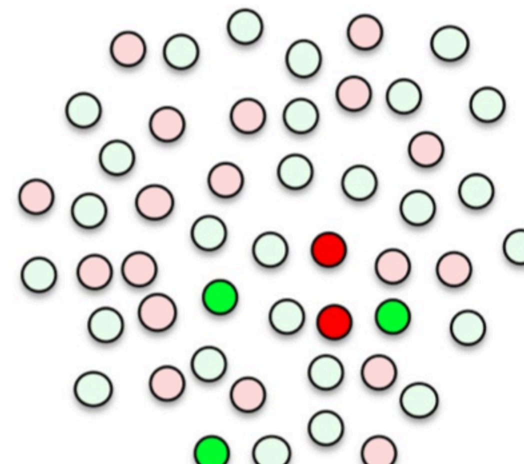

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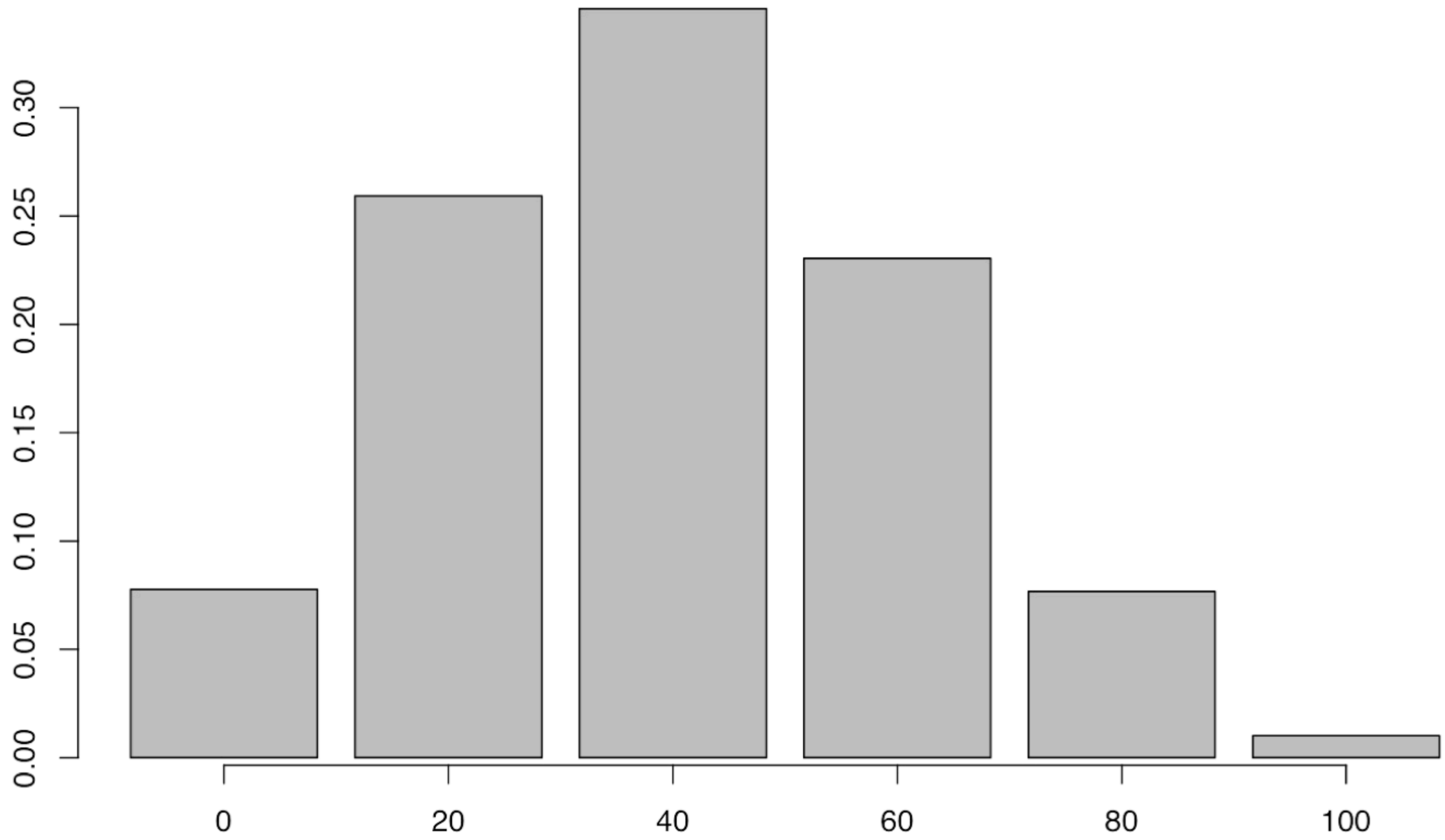
Example 1: Sampling

If you consider all 2,118,760 combinations, you'll get these answers with this distribution:



Example 1: Sampling

What if we sampled from the entire US population, instead of just 50 people?

- The number of combinations is huge: too large for most calculators

You can’t create a distribution of all possible combinations like in the previous example, but there are other statistics to approximate it (later in the semester)

Example 2: Password Security

Suppose you construct a password using letters and numbers (no symbols, for simplicity)

- How many potential passwords of length 6?
  - $36^6 = 2{,}176{,}782{,}336$
  - Or: 2.1×10^9
- Length 12?
  - 4.7×10^{18}
- Length 20?
  - 1.3×10^{31}

Example 2: Password Security

Suppose a hacker can guess 100,000 different passwords per second

Passwords of length 6:

(36^9) / 100,000 = 21768 seconds (~6 hours)

Passwords of length 20:

(36^20)/100,000 =
~37,000,000,000,000,000,000,000 hours
= 42,000,000,000,000,000 centuries

Example 2: Password Security

Sum rule:

How many passwords between length 6 and 8?

$36^6 + 36^7 + 36^8$

Example 2: Password Security

Suppose you construct a password by putting four English words together. How many possible passwords?

$30{,}000^4 = 8.1 \times 10^{18}$

- About the same as random letters/numbers of length 12

Example 3: Lotteries

In a lottery you have to guess 6 out of 49 numbers, in any order.
What are your chances of winning?

49-choose-6 = 13,983,816

Answer: about 1 in 14 million