PERMUTATIONS AND COMBINATIONS

Finite Mathematics for Data Science

- Statistics
- Probability
- Set Theory
- Combinatorics

‘Combination’ in Everyday Language

- Everyday language not precise about the meaning of the word ‘combination’
  - *“The soup includes a combination of fish and seafood items”*
    - Order in which the ingredients are listed does not matter.
  - *“The combination of the lock is 6-16-28.*
    - Order does matter here.

Math has precise meanings

- When order does not matter -> combination.
- When order does matter -> permutation.
- Permutations and combinations are closely connected – as are the formulas for calculating them.
- Always more permutations than combinations.
  - *1 combination of a,b,c.*
  - *6 permutations of a,b,c:*
    - abc, acb, bac, bca, cab, cba [no repetition allowed]
- Easiest to look at permutations first; then at combinations

Permutations with Repetitions

- How many permutations with repetition allowed when we are making a 3 letter permutation from a set with 5 elements: a, b, c, d, e
- aaa, aab, aac, aad, aae, aba, abb, abc, …
- While you could list them all, you could also reason about how many there are:
  - *First letter, there are 5 choices.*
  - *Second letter, there are 5 choices*
  - *Third letter there are 5 choices.*
- Hence, there are 5x5x5 = 125 choices.
- More generally, choosing r of something with n different types [where order matters and repetitions are allowed, n x n x n x … x n (r times = n^r

Examples of permutation with repetitions

- How many three number combinations in a lock are there if the possible numbers are the digits 0 through 9?
- 10 x 10 x 10 = 1000
- Cars sometimes have locks with four-digit codes but the choices of numbers are 0/1 2/3 4/5 6/7 8/9
  - *In this case it is 5x5x5x5 = 625*

Permutation without repetition

- How many permutations could a rack of pool balls be in?
  - *Numbers 1 through 15, plus a white (cue) ball (think of it as 0)*
  - *Without repetition, our choices get reduced each time.*
  - *Permutation without repetition 16 choices for first one, 15 choices for second one, etc.*
  - *Permutations = 16x15x14x…x3x2x1 = 16! (more than 20 trillion)*
- Suppose you only want 3 ball permutation without repetition
  - *16x15x14 = 3360*
- Factorial symbol n! = n x (n-1) x (n-2)x…x 2 x 1
- Special cases: 1! = 1, 0! = 1

General Rule of P(n,r)

- If only choosing 3 from the group of 16 balls, the permutations without repetition is 16 x 15 x 14
- But this can be written as 16!/13!
- General rule of permutations without repetition:

The number of permutations without repetition where n is the number of things to choose from, and r is the number of items we are choosing is given by

$$n! / (n-r)!$$

Example P(n,r)

- How many ways can first and second place be awarded in a race of 10 contestants?

  $P(10,2) = 10! / (10 – 2)! = 3628800 / 40320 = 9$

  $= 10 \times 9$

Combinations

- Also two types
  - *Repetition allowed (change in your pocket)*
  - *Repetition not allowed (lottery numbers)*
- Start with combinations without repetition (easiest to explain)
- Way to do the analysis
  - *First, do as a permutation problem (where order matters)*
  - *Second, alter the answer to get rid of the concern about order.*

Combinations C(n,r)

- Choose three balls from 16 (no repetition)
- We did that already and got 16! / (16 – 3)!
- Second, we divide by 3! Because there are 3! ways in which 3 balls can be ordered
- Then the number of combinations of 3 balls taken from 16, without repetition, is
- [16! / (16 – 3)!] / 3! = 16 x 15 x 14 / 3 x 2 x 1 = 560
- General formula for C(n,r) = n!/r!(n-r)!
- Expressed as ”n choose r”

Symmetry

- C(n,r) = C(n-r,r)
- Work out the formulas

The hard case: combinations with repetition

- Will not motivate it:
- $C_{rep}(n,r) = [r+n-1]!/r![n-1)!$
- Example: how many variations of triple scoop of ice cream with the ice cream flavors banana, chocolate, lemon, strawberry, vanilla
- $[3+5-1]!/3![5-1]! = 7!/[3! X 4!] = 5040/[6x24] = 35$