Rate of Change
Part 3: Derivatives

INFO-1301, Quantitative Reasoning 1

University of Colorado Boulder

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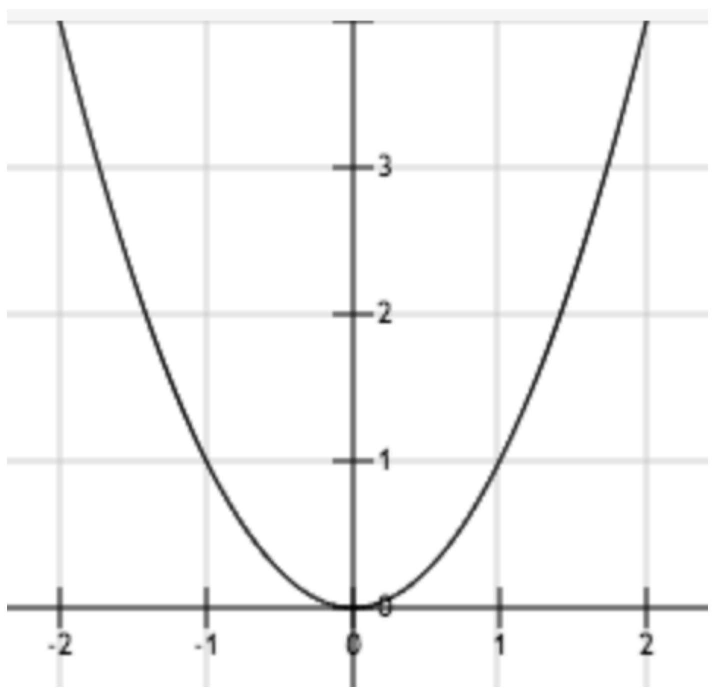
Slope for Nonlinear Functions

We've defined slope for straight lines (linear functions)

What about other kinds of functions?

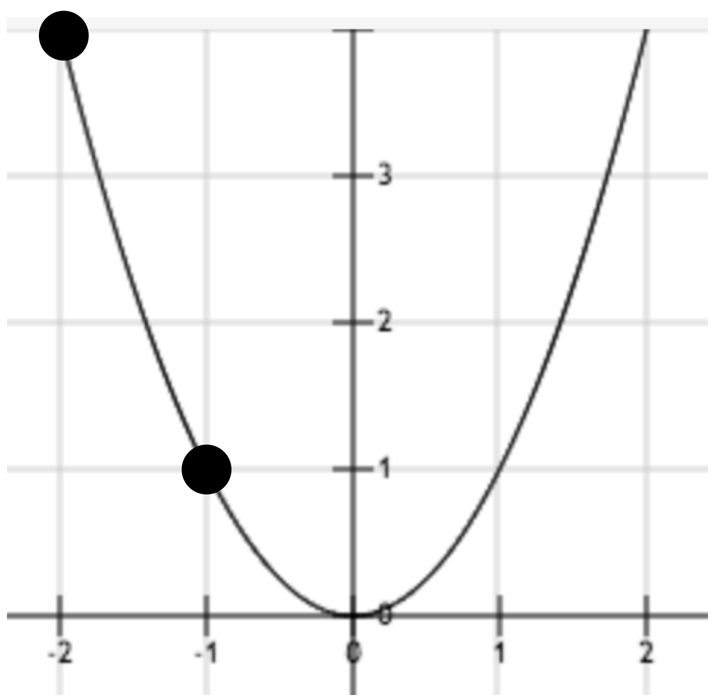
$f(x) = x^2$



Slope for Nonlinear Functions

For nonlinear functions, the “rise over run” formula gives you the **average slope** between two points

$f(x) = x^2$

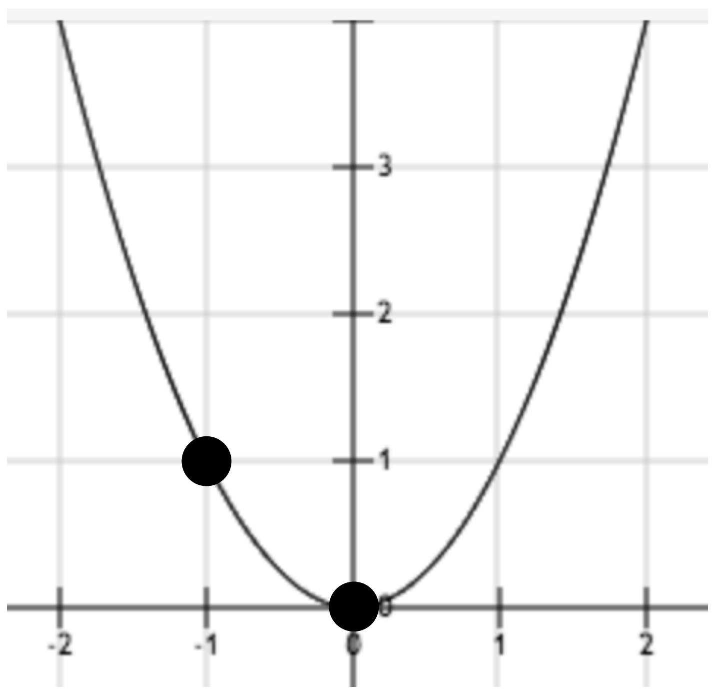


Average slope from x=-2 to x=-1 is:
-3

Slope for Nonlinear Functions

For nonlinear functions, the “rise over run” formula gives you the **average slope** between two points

$f(x) = x^2$

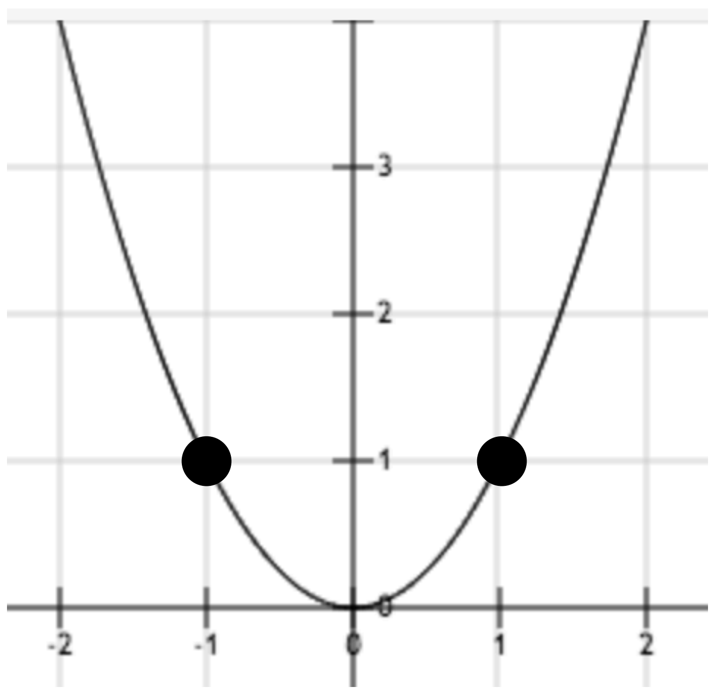


Average slope from x=-1 to x=0 is:
-1

Slope for Nonlinear Functions

For nonlinear functions, the “rise over run” formula gives you the **average slope** between two points

$f(x) = x^2$



Average slope from x=-1 to x=1 is:
0

Practice

A climber is on a hike. After 2 hours he is at an altitude of 6400 feet. After 6 hours, he is at an altitude of 6700 feet.

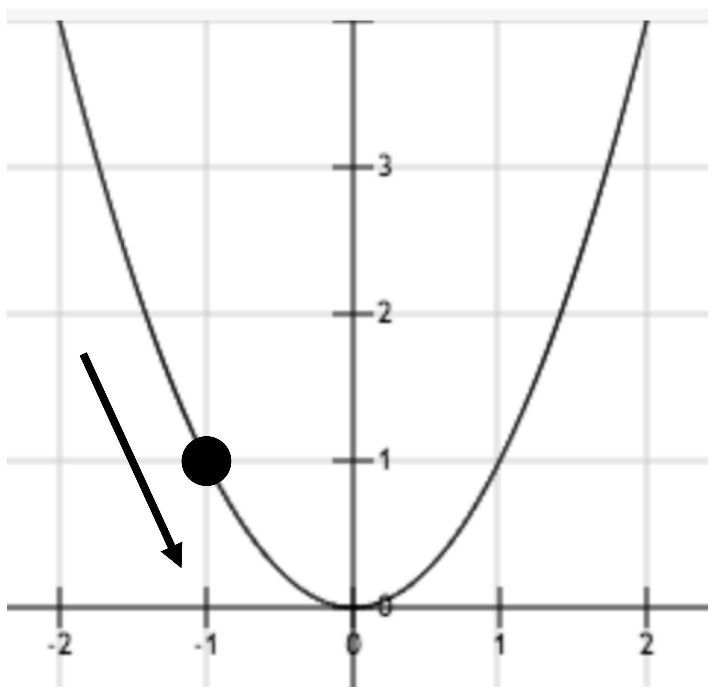
What is the average rate of change?

You can reason about the *average* rate of change without making any assumptions that the rate of change was the same during the entire duration

Slope at a Point

There is also a concept of slope for individual points (rather than two points)

$f(x) = x^2$

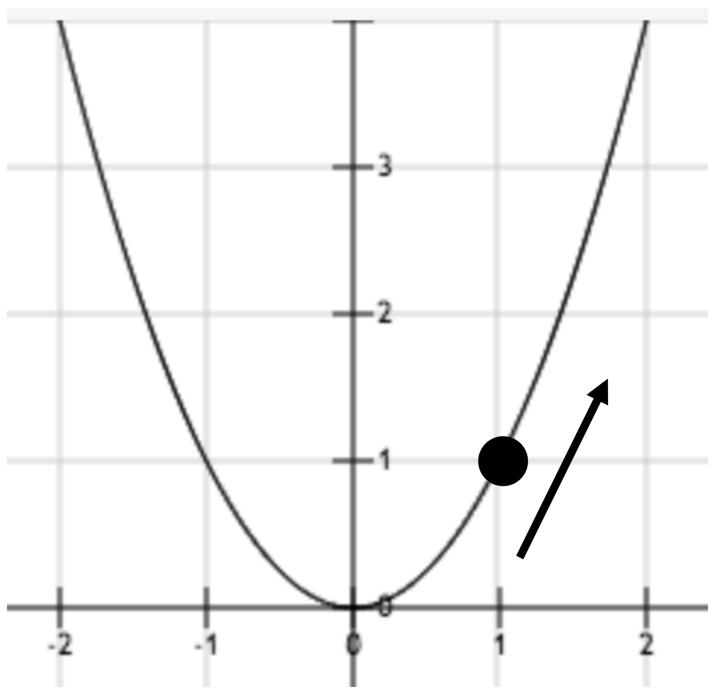


Slope at x=-1 is:
-2

Slope at a Point

There is also a concept of slope for individual points (rather than two points)

$f(x) = x^2$

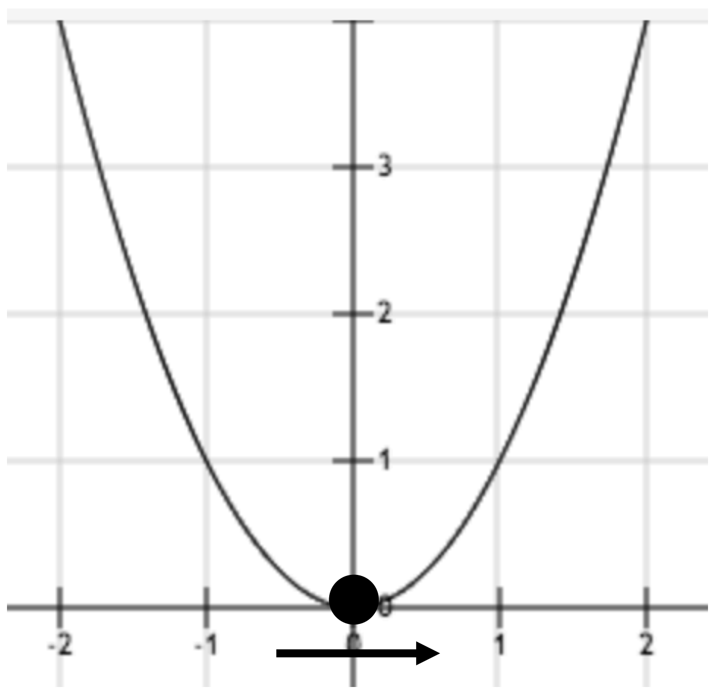


Slope at x=1 is:
2

Slope at a Point

There is also a concept of slope for individual points (rather than two points)

$f(x) = x^2$

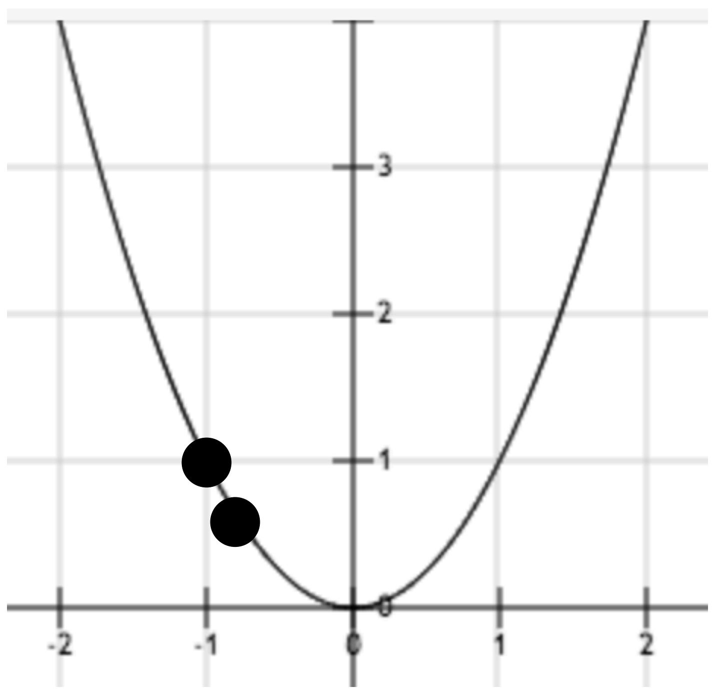


Slope at $x=0$ is:
0

Slope at a Point

The slope at a point is called the **derivative** at that point

$f(x) = x^2$



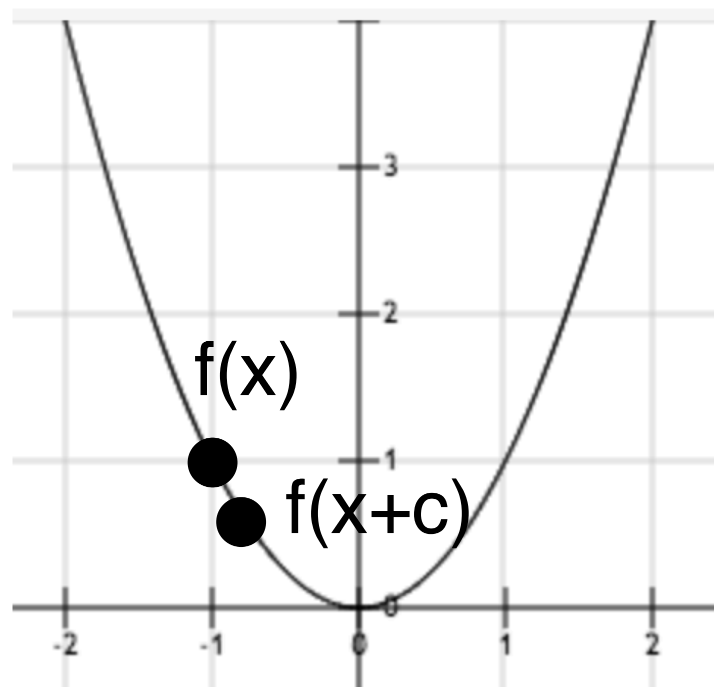
Intuition:
Measure the slope between two points that are really close together

Derivatives

The slope at a point is called the **derivative** at that point

Intuition: Measure the slope between two points that are really close together

$$\frac{f(x + c) - f(x)}{c}$$



Make *c* as small as possible (even zero!)

Derivatives

The derivative of x^2 is $2x$

Other ways of writing this:

$f'(x) = 2x$

$d/dx\ [x^2] = 2x$

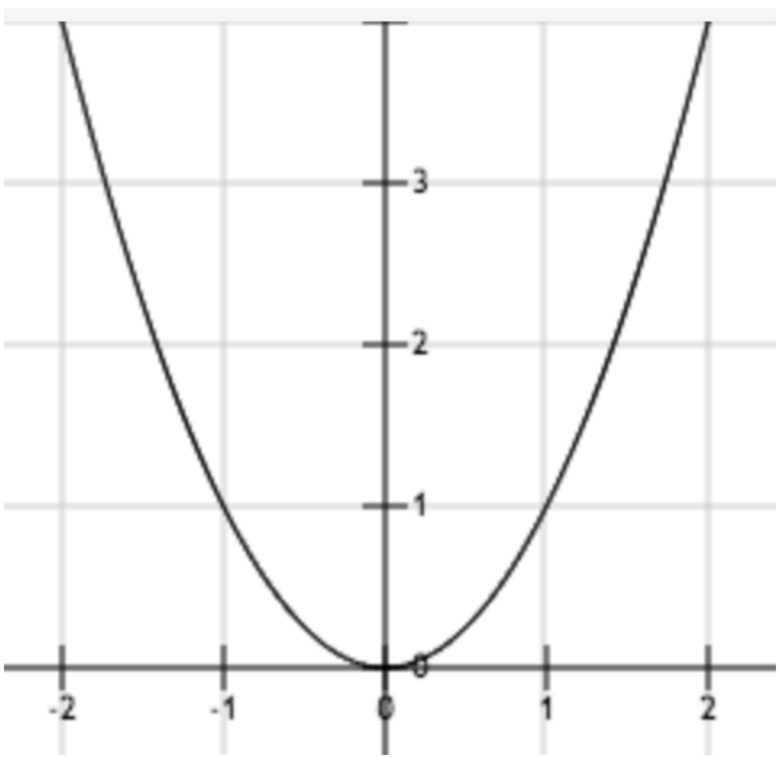
The derivative is also a function! It depends on the value of x.

- The slope is different at different points

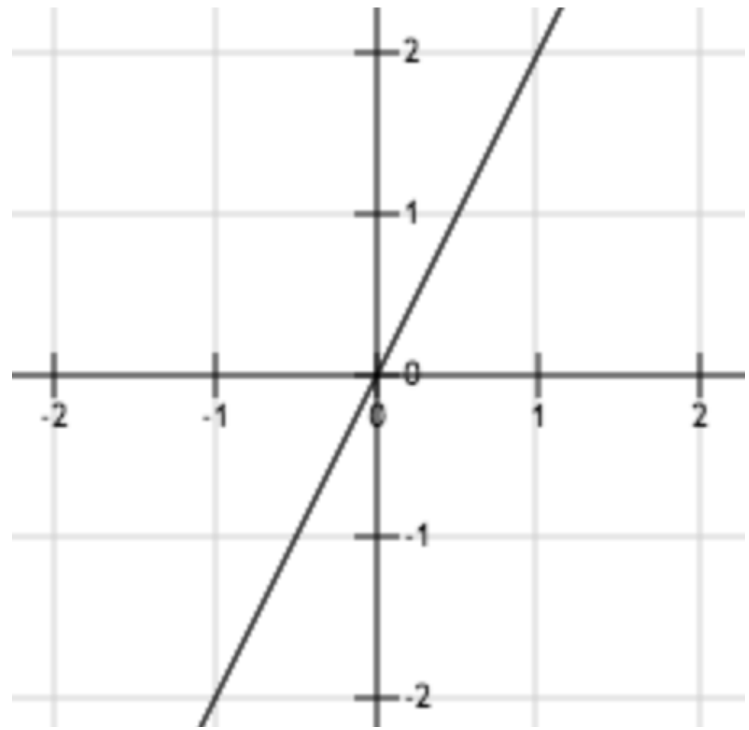
Derivatives

The derivative of x^2 is $2x$

f(x)



f'(x)



Calculating Derivatives

The derivative of a quadratic function is linear

The derivative of a linear function is constant

- Just the definition of slope you have seen before

The derivative of a constant is 0

The derivative of a sum of terms is the sum of the each derivative of each term

Calculating Derivatives

Constant rule:	$d/dx[c\,f(x)]$	$= c\,f'(x)$
Power rule:	$d/dx[x^n]$	$= n\,x^{n-1}$
Sum rule:	$d/dx[f(x) + g(x)]$	$= f'(x) + g'(x)$
Difference rule:	$d/dx[f(x) - g(x)]$	$= f'(x) - g'(x)$

The rules can be combined:

- e.g., constant and power: $c\,x^n = c\,n\,x^{n-1}$

Calculating Derivatives

Examples:

$d/dx[2x] = 2$

$d/dx[2] = 0$

$d/dx[2x + 5] = 2$

$d/dx[x^2 + 2x] = 2x + 2$

$d/dx[2x^2 + 2] = 4x$

$d/dx[2x^2 + 6x + 3] = 4x + 6$

Practice:

$d/dx[-x^2 + x – 10] = -2x + 1$

$d/dx[x^3] = 3x^2$

$d/dx[2x^3 + x^2 – 2x] = 6x^2 + 2x – 2$

Calculating Derivatives

Software to calculate derivatives:

Wolfram Alpha (wolframalpha.com)

Enter the query:

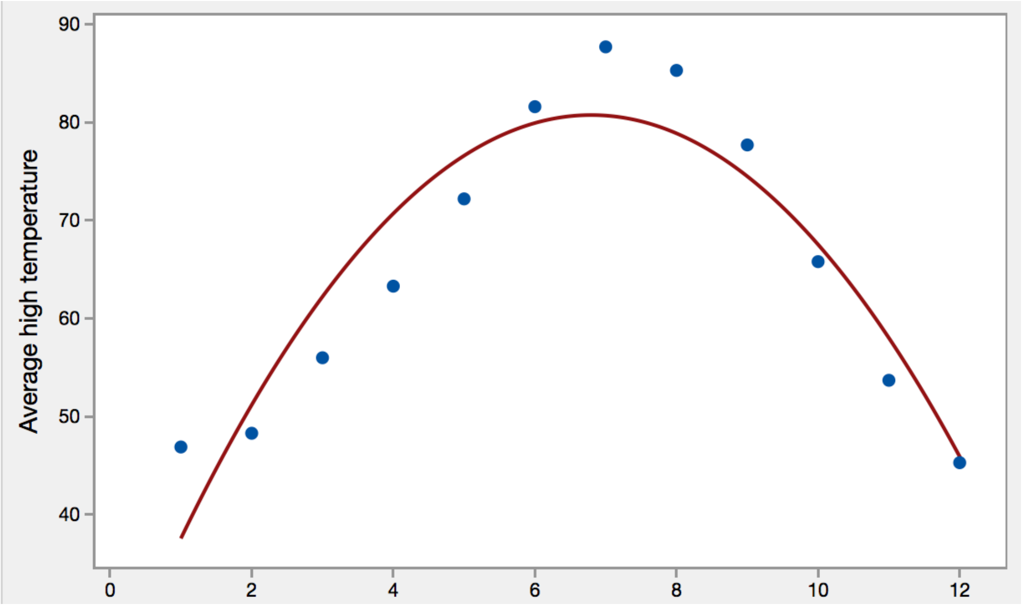
d/dx x^2 (or other functions)

Practice

The monthly temperature in Boulder can be approximated with a quadratic function:

$y = -1.3x^2 + 17.5x + 21.4$

What is the rate of change in temp. at each month?



Maxima and Minima

A local maximum is when the rate of change switches from positive to negative

A local minimum is when the rate of change switches from negative to positive

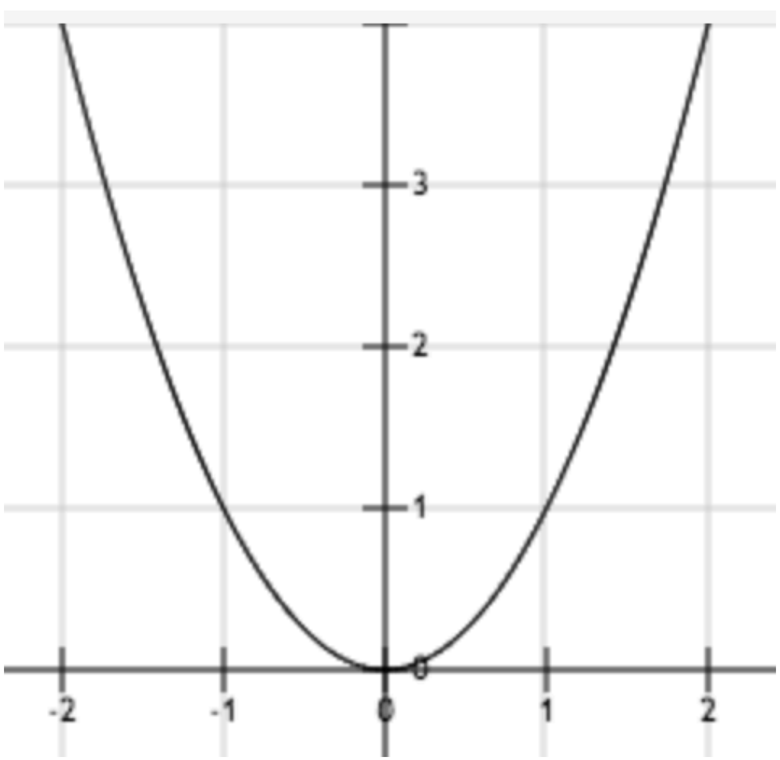
If the derivative is 0, there is a maximum or minimum at that point

To find when a function is maximized or minimized, set the derivative to 0 and solve for x

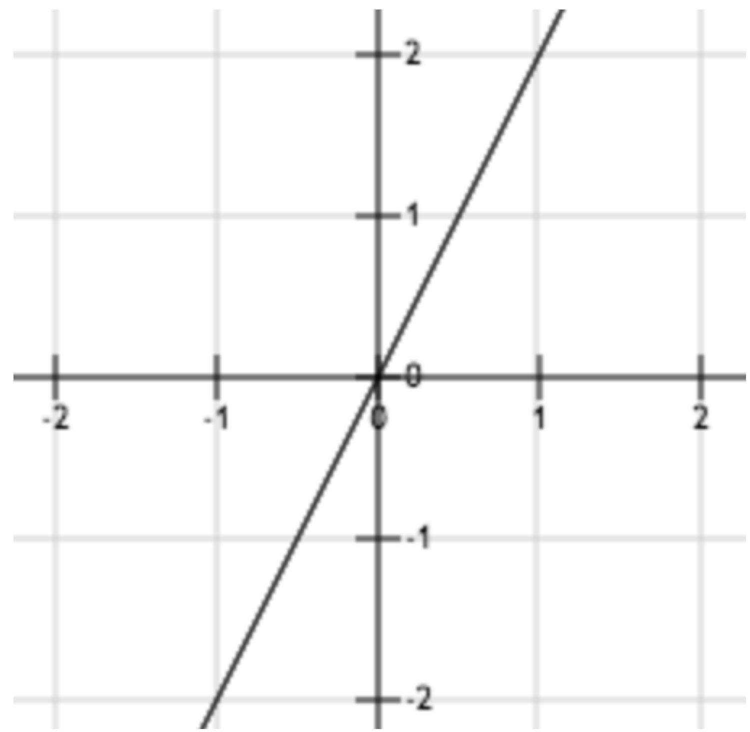
Maxima and Minima

Minimum (local and global) at **x=0**

f(x)



f'(x)



Practice

A recreational swimming lake is treated periodically to control harmful bacteria growth. Suppose *x* days after treatment, the concentration of bacteria per cubic centimeter is given by:

$f(x)=30x^2 - 240x + 500$

How many days after treatment will the concentration be minimal?

What is the minimal concentration?

Practice

A company estimates that its daily total cost function (based on number of items produced) is

$C(x) = x^3 - 6x^2 + 13x + 15$

and its total revenue function is

$R(x) = 28x$

Find the value of x that maximizes the daily profit.