Probability Basics
Part 2: Probability Operations

INFO-1301, Quantitative Reasoning 1

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Operations

Last month we learned about different mathematical *operations* for sets and booleans:

Sets:	**Booleans:**
Intersection	AND
Union	OR
Complement	NOT

These operations can also be used to compute probabilities for random variables

Example

Consider the probability of different outcomes from the roll of a die

$$\underbrace{\begin{aligned} P(X=1) &= 1/6 \\ P(X=2) &= 1/6 \\ P(X=3) &= 1/6 \\ P(X=4) &= 1/6 \\ P(X=5) &= 1/6 \\ P(X=6) &= 1/6 \end{aligned}}_{\text{Distribution over 6 possible outcomes}}$$

A distribution where all outcomes are equally likely is a **uniform** distribution

Disjunctions

If two or more outcomes cannot all be true at once, they are called **disjoint** or **mutually exclusive**

Die roll outcomes are disjoint

- A die cannot land a 3 and also a 4

Complements

The **complement** of an outcome is the set of all other outcomes in the sample space

- This is the same as the set complement operation that you learned about before

Remember:

Sample space is the *domain* of the random variable, which is a *set* of the possible outcomes

When discussing complements, we assume the outcomes are disjoint

AND

The probability that multiple outcomes are true can be described with an AND expression

$P(X\!=\!3 \text{ AND } X\!=\!4) = 0$

If the outcomes are disjoint, the probability of the AND of multiple outcomes will always be 0

AND

The probability that multiple outcomes are true can be described with an AND expression

Harder example: two dice
X is outcome of first die; Y is outcome of second

P(X=3 AND Y=4) = 1/36
P(X=4 AND Y=3) = 1/36
P(X=4 AND Y=4) = 1/36
…

OR

The probability that *any* outcome is true can be described with an OR expression

P(X=3 OR X=4) =   2/6

Addition rule:
If outcomes are disjoint, the probability that any of them are true is the *sum* of their individual probabilities

OR

The probability that *any* outcome is true can be described with an OR expression

$$\begin{aligned} P(X > 3) &= P(X{=}4 \text{ OR } X{=}5 \text{ OR } X{=}6) \\ &= P(X{=}4) + P(X{=}5) + P(X{=}6) \\ &= 1/6 + 1/6 + 1/6 \\ &= 1/2 \end{aligned}$$

OR

What if the outcomes aren't disjoint?

Harder example: two dice
X is outcome of first die; Y is outcome of second

P(X=3 OR Y=4) = ?

P(X=3) + P(Y=4) isn't quite right:
the outcome X=3 AND Y=4 is counted twice

OR

What if the outcomes aren't disjoint?

Harder example: two dice
X is outcome of first die; Y is outcome of second

$P(X=3 \text{ OR } Y=4) =$

$P(X=3) + P(Y=4)$

$- P(X=3 \text{ AND } Y=4)$ ⬅ Subtract out the AND which is double counted

OR

What if the outcomes aren't disjoint?

General addition rule: (for two outcomes)
The probability that either outcome is true is the *sum* of their individual probabilities, *minus* the probability that they are both true

- i.e., $P(X \text{ OR } Y) = P(X) + P(Y) - P(X \text{ AND } Y)$

Similar to calculating the cardinality of a set union:
$|A \cup B| = |A| + |B| - |A \cap B|$

NOT

The probability that an outcome is *not* true is the probability of any other outcome in the sample space

P(X is NOT 3)

= P($X \neq 3$)

= P(X=1 OR X=2 OR X=4 OR X=5 OR X=6)

= 5/6

= 1 – P(X=3)

NOT

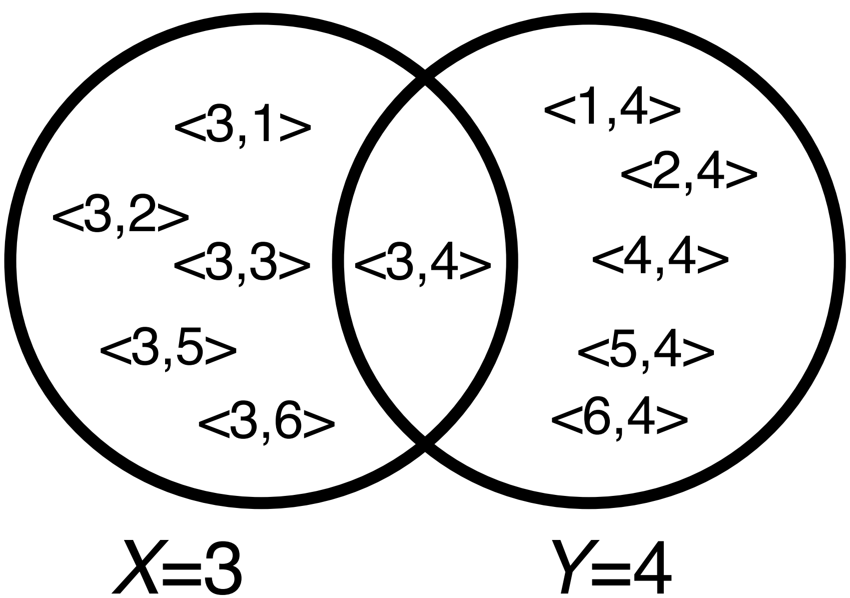
The probability of the complement of an outcome is always 1 minus the probability of the outcome

$$P(\text{NOT } X) = 1 - P(X)$$

Venn Diagrams

If you have multiple outcomes you can draw the relationships between them as a Venn diagram

- That is, draw the sets of the outcomes that correspond to what you are calculating



The *intersection* is X=3 AND Y=4
The *union* is X=3 OR Y=4

The *complement* of X=3 is X≠3

- We assume the universal set is all possible dice rolls